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CERTIFICATION COURSE

STATE-SPACE METHOD

STATE SPACE REPRESENTATION

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FREQUENCY DOMAIN Vs STATE-SPACE

The STATE-SPACE approach is a modern, time domain approach and a unified method for modelling, analyzing and designing a wide range of systems.

Frequency Domain Approach

- Applicable to only LINEAR systems
- TIME-INVARIANT systems
- Single Input- Single Output systems

State-Space Approach

- Applicable to NONLINEAR systems
- TIME VARYING systems, e.g. missiles
- Systems with non-zero initial conditions
- Multiple Input- Multiple Output systems
- Digital computers, digital simulations

The STATE-SPACE approach is not as intuitive as the classical approach. The designer has to engage in several calculations before the physical interpretation of the model is apparent.



STATE-SPACE REPRESENTATION

- **System variable:** any variable that responds to an input or initial conditions in a system
- **State variables:** the smallest set of linearly independent system variables such that the values of the members of the set at time t_0 along with known forcing functions completely determine the value of all system variables for all $t \geq t_0$.
- **State vector:** a vector whose elements are the state variables
- **State space:** the n -dimensional space whose axes are the state variables
- **State equations:** a set of n simultaneous, first-order differential equations with n variables, where the n variables to be solved are the state variables
- **Output equations:** the algebraic equation that expresses the output variables of a system as linear combinations of the state variables and the inputs

STATE-SPACE REPRESENTATION

- A system is represented in state space by the following equations

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}\end{aligned}$$

- $\mathbf{x}$ = state vector
- $\dot{\mathbf{x}}$ = derivative of the state vector with respect to time
- $\mathbf{y}$ = output vector
- $\mathbf{u}$ = input or control vector
- $\mathbf{A}$ = system matrix
- $\mathbf{B}$ = input matrix
- $\mathbf{C}$ = output matrix
- $\mathbf{D}$ = feed forward matrix

REFERENCE BOOKS

- Norman S. Nise, Control Systems Engineering, Wiley, 2013
- Katsuhiko Ogata, Modern Control Engineering, Prentice Hall, 2010.

